

Pattern Synthesis of Non-uniform Array using Weighted Least Square Algorithm

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Abstract— The least square algorithm is often used for the pattern synthesis of uniform phased arrays due to its fast convergence owing to exact matrix-based formulation. This paper presents the pattern synthesis of linear array with non-uniform inter-element spacing using the weighted least square algorithm. The computed results demonstrate that the algorithm has direct control over the desired mainlobe width, the sidelobe level, the position and width of the notch even when the spacing between the antenna elements is non-uniform or random.

I. INTRODUCTION

The analytical weighted least square algorithm defines the problem as a set of linear equations, which when solved for a given excitation, produces a desired far field pattern. Since the formulation is based on the exact matrix equations, the optimal solution is obtained within few iterations. Carlson and Willner [1] employed the method of weighted least squares in pattern synthesis by segregating the real and imaginary parts of the electric field, and the complex excitation vectors. This approach doubles the dimension of all the matrices corresponding to the desired far-field, excitation field, and weights. The inversion of such large real matrices becomes computationally complex. In the work presented here, instead of taking real and imaginary components of matrices separately, the complex form of fields is considered for the estimation of the weight vector. This method simplifies the computations for the excitation vector required to generate the desired pattern. In the work reported here, weighted least square algorithm is used to generate pattern with desired mainlobe width for non-uniform linear arrays. The role of parameters viz., the level of weighting in different regions of antenna pattern, number of elements, and inter-element spacing is analyzed. It is shown that the algorithm has a direct control over the placement of notch within the sidelobe region of the pattern, beam steering, and the mainlobe width. The algorithm employed proves to be very simple, fast, and flexible as compared to other algorithms being proposed for non-uniform array [2], [3].

II. PATTERN SYNTHESIS OF NON-UNIFORM ARRAY

The inter-element spacing varies continuously as one move away from the centre of the array, i.e. $d_n = d + \text{space}$. Here d_n is

the element position from the center of the array, d is the first element position from the center of the array, and space is the incremental increase in the spacing between the adjacent antenna elements. The three cases considered for N-element non-uniform array are as follows:

- Case 1: $\text{space} = 0.12n$, where $n = 0, 1, 2, \dots N/2$
- Case 2: $\text{space} = 0.1n$, where $n = 0, 1, 2, \dots N/2$
- Case 3: Random spacing with length of array equal to 8λ

In case of non-uniform spacing between the antenna elements, the steering vector takes the form,

$$U(\theta) = [1 \quad e^{-j\Phi} \quad e^{-2j\Phi} \quad \dots \quad e^{-j(N-1)\Phi}]^T \quad (1)$$

$$\text{where, } \Phi = \frac{2\pi\{S(p+1, 1) - S(p, 1)\}}{\lambda} \sin \theta; p = 1, 2, \dots N \quad (2)$$

Here, S is the position of the element along the axis.

The transformation matrix H relates the excitation vector to the field vectors as

$$F(\phi) = H(\phi)X \quad (3)$$

$$H = \cos \theta + j \sin \theta; \quad \theta = \frac{2\pi}{\lambda} S(p, 1) \times \sin \phi$$

In matrix form, $(F)_{m \times 1} = (H)_{m \times n} \times (X)_{n \times 1}$ where, m is the number of field points and n is the number of antenna elements. The cost function representing the difference between the actual field and the desired field is given by

$$J = (D - HX)^T W (D - HX) \quad (4)$$

where W is a diagonal weighting matrix used to generate the desired field pattern. For the minimum cost function, the excitation vector is obtained as [1]

$$X = (H^T W H)^{-1} H^T W D \quad (5)$$

If the weight vector is proper, this excitation vector will provide the desired field pattern,

$$E = (X^*)^T \cdot U \quad (6)$$

where U is the steering vector and superscript T denotes the hermitian.

III. RESULTS AND DISCUSSION

The radiation pattern of a 16-element linear array with non-uniform spacing (Case 1) is shown in Fig. 1. It can be seen

that the algorithm generates the pattern efficiently with the desired main lobe width (-5° to $+5^\circ$). Next the pattern of same non-uniform array is compared with that of uniform array with the mainlobe width (-10° to $+10^\circ$), shown in Fig. 2. Unlike uniform array pattern, the sidelobe level of non-uniform array is significantly lower, and instead of flat top, pointed mainlobe is generated maintaining the desired mainlobe width.

Fig. 3 presents the radiation pattern of 16-element non-uniform array (Case 2) with the main beam steered at 10° and notch in the region from $+30^\circ$ to $+50^\circ$. It can be inferred that the weighted least square algorithm is efficient in controlled beam steering and placing the notch in the desired sidelobe region of the pattern. The depth of the notch can be controlled by proper weighting in the desired region. Fig. 4 shows the comparison of the radiation patterns for non-uniform (Case 3) and uniform array with equal length and equal number of antenna elements. It can be seen that the sidelobe distribution in the two patterns is very different, even when compared with Case 1 (Fig. 1), where only the non-uniform spacing is considered and the constraint of equal length is waived off. Further it is observed that since the size of uniform and non-uniform array is same, the mainlobe width of the pattern remained same.

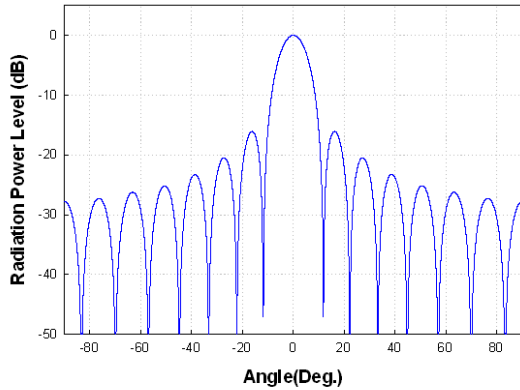


Fig. 1. Pattern of 16-element non-uniform linear array (Case 1). Main lobe= -5° to $+5^\circ$

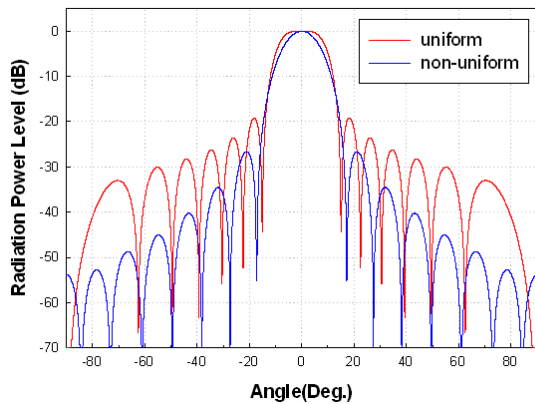


Fig. 2. Comparison of patterns of non-uniform (Case 1) and uniform linear array. $N=16$, main lobe= -10° to $+10^\circ$

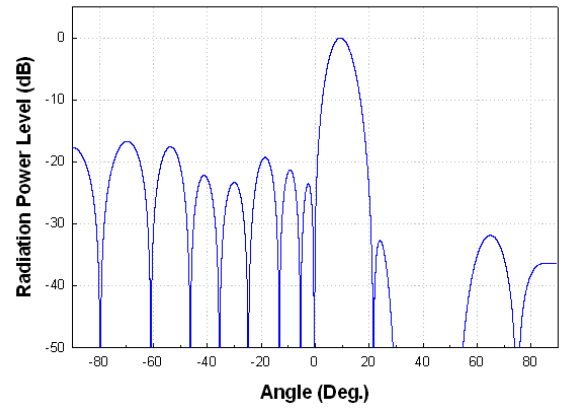


Fig. 4. Pattern of 16-element non-uniform array (Case 2) with notch ($+30^\circ$ to $+50^\circ$). Main beam steered at 10° .

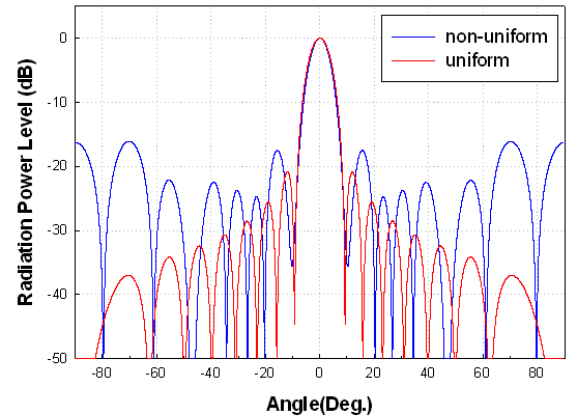


Fig. 3 Comparison of patterns of non-uniform and uniform linear array. $N=16$, main lobe= -5° to $+5^\circ$, length of array= 8λ .(Case 3)

IV. CONCLUSION

The efficiency, flexibility and fast convergence of the weighted least square algorithm is demonstrated for the pattern synthesis of non-uniform arrays. It is shown that the radiation pattern with low sidelobe level can be obtained with non-uniform/random inter-element spacing. The algorithm has complete control over the main lobe width, sidelobe level, beam steering and placing a deep notch in the desired sidelobe region. This study shows that the algorithm employed can be exploited for active cancellation of the probing sources provided direction-of-arrival of the signals is known *a priori*.

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